

ECS 452 Additional Examples for Section 5.1

1. Suppose the generator matrix of a linear code is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$g^{(1)}$
 $g^{(2)}$
 $g^{(3)}$

a. Find the codeword for the message $\underline{b} = [1 \ 0 \ 0]$

$$\begin{aligned} \underline{x} &= \underline{b} \mathbf{G} = [1 \ 0 \ 0] \begin{bmatrix} g^{(1)} \\ g^{(2)} \\ g^{(3)} \end{bmatrix} = (1 \cdot g^{(1)}) \oplus (0 \cdot g^{(2)}) \oplus (0 \cdot g^{(3)}) = g^{(1)} \\ &= [1 \ 0 \ 0 \ 1 \ 0 \ 1] \end{aligned}$$

b. Find the codeword for the message $\underline{b} = [0 \ 1 \ 1]$

$$\underline{x} = g^{(2)} \oplus g^{(3)} = [0 \ 1 \ 1 \ 1 \ 0 \ 1]$$

2. Suppose the generator matrix of a linear code is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

+

a. Find the codeword for the message $\underline{b} = [1 \ 0 \ 0 \ 0]$

$$[1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0]$$

b. Find the codeword for the message $\underline{b} = [0 \ 1 \ 1 \ 0]$

$$[1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0]$$

3. Suppose the generator matrix of a linear code is given by

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Find the complete codebook of this code.